

# Are Galaxies Cosmon Lumps?

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## Abstract

The scalar “cosmon” field mediating quintessence influences the dynamics of extended objects in the universe. We discuss cosmon lumps – spherically symmetric solutions for the scalar field coupled to gravity. The two integration constants can be associated to the mass and the rotational velocity in a halo-like region with constant rotation curve. The presence of the scalar field also changes the singularity of the black hole solution. We ask if galaxies could be associated with cosmon lumps.

Quintessence is an interesting and perhaps the most natural candidate for the dark energy in the universe [1], [2]. It is mediated by a scalar field – the cosmon – which is driven by a decaying potential to infinity. The dark energy of the universe is associated to the potential and kinetic energy of the coherent motion of the cosmon field and therefore homogeneously distributed in the universe [1], [2]. For an appropriate form of the effective cosmon potential and kinetic term the equation of state of quintessence may have a negative pressure today. This could explain many of the recently observed surprising features of the universe [3]. Quintessence influences the age of the universe [1],[2], the effective equation of state of the energy momentum tensor [1],[2], the formation of structure [2], [4], [5], [6] and the detailed fluctuation pattern of the cosmic microwave background [2],[4],[7],[8]. A negative pressure in the present epoch reconciles a large amount of dark energy with structure formation and explains the cosmological acceleration as observed from distant supernovae [9].

All this concerns the role of the homogeneous field which can be interpreted as the space average of the cosmon field. The cosmon is essentially massless on scales smaller than the horizon [1]. Therefore, there are also important questions about the role it could play on the scale of galaxies and clusters. Could the incoherent cosmon fluctuations

on small scales play the role of dark matter [10] and drive the formation of structure in the universe? Which role plays the cosmon field in a galaxy or a cluster? If the cosmon has a weak coupling to baryonic matter or to some unknown particle constituting the dark matter [1], the local cosmon fluctuation has necessarily a nonzero value in any concentration of matter. In this case a source term drives the local field away from the cosmological value [11]. Even without such a coupling the presence of a local cosmon field in extended objects seems quite plausible. Due to its long-range character this field would not vanish outside a matter concentration. The cosmon coupling to gravity modifies then the gravitational solutions for “empty space” around local objects.

We speculate in this note that the cosmon could even dominate the energy momentum tensor of extended objects. If so, one may in a first approximation neglect the matter component and investigate the coupled system of cosmon and gravity. More generally, the “matter” (e.g. baryons) of an extended object may be concentrated in an inner “bulk region”. Outside of the bulk the solution of the cosmon-gravity field equations becomes relevant for all models with quintessence. We discuss\* spherically symmetric solutions with intriguing properties resembling galactic halos with a constant rotation curve. The outer region of galaxies could be associated to such a cosmon lump, whereas the inner region needs the inclusion of matter. In a more conservative scenario dark matter would be an essential component of the halo. Still, the cosmon field may play a quantitatively important role. In particular, its presence modifies the region around the central singularity.

For the standard isotropic metric  $ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$  the gravitational field equations in presence of a scalar field  $\varphi(r)$  are given by

$$\begin{aligned} R_{00} &= \frac{B''}{2A} - \frac{B'}{4A} \left( \frac{A'}{A} + \frac{B'}{B} \right) + \frac{B'}{rA} = -\frac{BV}{2M^2} \\ R_{rr} &= -\frac{B''}{2B} + \frac{B'}{4B} \left( \frac{A'}{A} + \frac{B'}{B} \right) + \frac{A'}{rA} = \frac{AV + \varphi'^2}{2M^2} \\ R_{\theta\theta} &= 1 + \frac{r}{2A} \left( \frac{A'}{A} - \frac{B'}{B} \right) - \frac{1}{A} = \frac{r^2V}{2M^2} \end{aligned} \quad (1)$$

Here primes denote derivatives with respect to  $r$  and  $M^2 = M_p^2/16\pi = (16\pi G)^{-1}$ . For simplicity of the discussion we assume here that quintessence is characterized by a standard kinetic term and an exponential potential

$$V = M^4 \exp(-\alpha \frac{\varphi}{M}) \quad (2)$$

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\*For solutions of the Einstein-Klein-Gordon field equations in the context of Bose stars see ref. [12].

such that the scalar field equation reads

$$\varphi'' + \left( \frac{2}{r} - \frac{A'}{2A} + \frac{B'}{2B} \right) \varphi' = A \frac{\partial V}{\partial \varphi} = -\frac{\alpha}{M} AV \quad (3)$$

We will present a class of solutions for which the precise form of the potential term is actually not important for the qualitative behavior. Other potentials for quintessence may therefore be used as well, provided that they share the rough features characteristic for quintessence.

We concentrate first on a free scalar field with  $V = 0$ . Appropriate linear combinations of the field equations yield two coupled first-order differential equations for  $A$  and

$$w = \frac{r}{M} \varphi' \quad (4)$$

namely

$$rw' = -Aw, \quad rA' = -A(A - 1 - \frac{w^2}{4}) \quad (5)$$

The general solution depends on two integration constants  $R_s$  and  $\gamma$

$$w = \gamma \frac{R_s}{\rho}, \quad A = 1 + \frac{R_s}{\rho} - \frac{\gamma^2}{4} \left( \frac{R_s}{\rho} \right)^2 \quad (6)$$

and the new radial coordinate  $\rho$  is related to  $r$  by

$$\frac{\partial \ln \rho}{\partial \ln r} = A \quad (7)$$

It obeys

$$r = (\rho - \rho_H)^{\frac{1}{2}-\delta} (\rho + \rho_H + R_s)^{\frac{1}{2}+\delta} \quad (8)$$

with

$$\rho_H = (\sqrt{\gamma^2 + 1} - 1) \frac{R_s}{2}, \quad \delta = \frac{1}{2\sqrt{\gamma^2 + 1}} \quad (9)$$

The remaining field equations for  $B$  and  $\varphi$  are given by

$$\frac{\partial \ln B}{\partial \ln \rho} = \frac{R_s}{A\rho}, \quad \frac{\partial \varphi}{\partial \ln \rho} = \frac{\gamma M R_s}{A\rho} \quad (10)$$

and we note that a combination of  $\ln B$  and  $\varphi$  is independent of  $\rho$

$$\ln B - \frac{\varphi}{\gamma M} = \text{const} = -\frac{\varphi_\infty}{\gamma M} \quad (11)$$

For large  $r \gg \rho_H$  we observe the limits

$$\lim_{r \rightarrow \infty} \rho = r - R_s, \quad \lim_{r \rightarrow \infty} B = 1 - \frac{R_s}{r} + O(r^{-3}), \quad \lim_{r \rightarrow \infty} A = \left(1 - \frac{R_s}{r}\right)^{-1} - \frac{\gamma^2 R_s^2}{4r^2} \quad (12)$$

and we identify  $R_s$  with the Schwarzschild radius

$$R_s = 2mG = \frac{m}{8\pi M^2} \quad (13)$$

In lowest (nontrivial) order post-Newtonian gravity there is no deviation from the usual results of general relativity. In this respect the cosmon is quite different from the Jordan-Brans-Dicke theory [13], [1]. Furthermore, for  $\gamma \rightarrow 0$  one has  $\delta \rightarrow 1/2$ ,  $\rho_H \rightarrow 0$ ,  $r = \rho + R_s$  and our solution approaches the well-known Schwarzschild solution in empty space for all values of  $r$ .

The constant  $\gamma$  reflects the role of the scalar gradient energy density

$$\rho_{grad} = \frac{\varphi'^2}{2A} = \frac{M^2 w^2}{2r^2 A} = \frac{\gamma^2 R_s^2 M^2}{2r^2 \rho^2 A} \quad (14)$$

which decays for large  $r$  as  $\rho_{grad} \sim r^{-4}$ . We emphasize that the total mass of the object (as given by the integration constant  $R_s$ ) is not directly related to the scalar gradient energy density. Indeed, gravitational and scalar energy density are of comparable strength and may partially cancel. This effect allows two independent integration constants even for large  $\gamma$ . We also note that in linear approximation the Newtonian potential  $\sim g_{00}$  couples to  $S_{00} = T_{00} - \frac{1}{2}T_\rho^\rho g_{00}$  which vanishes for a free scalar field. In absence of a potential there is a simple mapping between solutions with  $\gamma > 0$  ( $\varphi$  decreasing for  $r \rightarrow 0$ ) and  $\gamma < 0$  ( $\varphi$  increasing for  $r \rightarrow 0$ ).

For  $\gamma \neq 0$  the scalar field generates a new characteristic length scale  $\rho_H$ . For  $r$  in the vicinity of  $\rho_H + R_s$  or smaller the overall behavior of our solution changes drastically as compared to the Schwarzschild solution. In particular, for  $r \ll \rho_H + R_s$  the approximate behavior ( $2/(1 - 2\delta) = 2 + R_s/\rho_H$ )

$$\rho = \rho_H + (2\rho_H + R_s)^{-(1 + \frac{R_s}{\rho_H})} r^{2 + \frac{R_s}{\rho_H}} \quad (15)$$

implies constant  $\rho(r) \rightarrow \rho_H$  for  $r \rightarrow 0$ . As a consequence, the Schwarzschild singularity at  $r = R_s$  has now disappeared. The only possible singular behavior can occur for  $r \rightarrow 0$  where

$$\begin{aligned}\lim_{r \rightarrow 0} A &= \left(2 + \frac{R_s}{\rho_H}\right)^{-\frac{R_s}{\rho_H}} \left(\frac{r}{\rho_H}\right)^{2+\frac{R_s}{\rho_H}} \\ \lim_{r \rightarrow 0} B &\sim \left(\frac{r}{\rho_H}\right)^{\frac{R_s}{\rho_H}}\end{aligned}\tag{16}$$

Even for arbitrarily small  $|\gamma| \neq 0$  the black hole solution gets modified!

An interesting situation arises for  $\rho_H \gg R_s$  which corresponds to  $\gamma^2 \gg 1$ . In this case we find a “halo region” with constant rotation curve for  $r \ll \rho_H$ . Indeed, in this region  $B$  evolves logarithmically

$$B(r) \approx 1 + \frac{2}{|\gamma|} \ln \frac{r}{\rho_H}\tag{17}$$

The velocity of objects in stable circular orbits at distance  $R$  from the center becomes therefore essentially constant

$$v_{rot}^2(R) = \frac{R}{2} \frac{\partial B(R)}{\partial R} \approx \frac{1}{|\gamma|}\tag{18}$$

resembling very much the flat rotation curves observed [14] in the galactic halos! Relating the radius of the halo to the total mass and the rotation velocity

$$\rho_H \approx \frac{|\gamma| R_s}{2} \approx \frac{R_s}{2v_{rot}^2} = \frac{mG}{v_{rot}^2} = \frac{m}{16\pi M^2 v_{rot}^2}\tag{19}$$

we find for a lump with  $m = 3 \cdot 10^{11} M_\odot$  and  $v_{rot} = 150$  km/sec a halo size of around 60 kpc, well compatible<sup>†</sup> with observed rotation curves of galaxies! These simple properties lead to the speculation that galaxies may be cosmon lumps, with the flat rotation curve arising through the interplay of gravitational and scalar field equations rather than from the usually assumed dark matter in the halo! Furthermore, for cosmon lumps generated in early cosmology one may assume an almost constant average density of mass/halo, typically characterized by a small power  $\zeta$ ,  $m\rho_H^{-3} \sim m^\zeta$ . This implies a scaling relation between  $|\gamma|$  and  $m$ , i.e.

$$m \sim v_{rot}^{\frac{6}{2+\zeta}}\tag{20}$$

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<sup>†</sup>We recall here that the mass estimates from cold dark matter models do not apply in our case since the flattening of the rotation curve is not due to pressureless matter. Otherwise, the currently estimated value of  $v_{rot} \approx 330$  km/sec for  $m = 3 \cdot 10^{11} M_\odot$  would lead to a too small halo  $\sim 12$  kpc.

and can fit the observation.

Before going on with the speculation that (some) galaxies are cosmon lumps, one should check if the unusual features of cosmon lumps are not in contradiction with observation. The most striking effect is perhaps the substantial deviation of  $A$  from one inside the halo<sup>‡</sup>

$$A \approx \frac{r^2}{\rho_H^2} \quad (21)$$

For local gravity measurements at some place  $R$  in the halo not too close to the singularity the resulting effects seem to be small. We can use a “cartesian” coordinate system  $ds^2 = -dt^2 + C(\tilde{\rho})d\vec{x}d\vec{x}$  with

$$C(\tilde{\rho}) = \frac{r^2}{\tilde{\rho}^2} \approx \frac{\rho_H^2}{\tilde{\rho}^2} \ln^2 \left( \frac{\tilde{\rho}}{\tilde{\rho}_c} \right), \quad \tilde{\rho} = \sqrt{\tilde{x}^2} \approx \tilde{\rho}_c \exp \left( \frac{r}{\rho_H} \right) \quad (22)$$

(where we have taken the limit  $1/|\gamma| = 0$ ,  $B = 1$ ). Choosing an appropriate scale for  $\tilde{\rho}$  with  $C(\tilde{\rho}(R)) = 1$  the deviations from the “local” Minkowski metric are of the order  $(\partial C/\partial \tilde{\rho})\Delta = O(\Delta\rho_H/(\tilde{\rho}(R)r(R))) = O(\Delta\rho_H/R^2)$  where  $\Delta$  is the typical length scale of the local observation. This should be compared to the local effect of a star on the space-like metric  $\sim 2GM_\odot/\Delta$  or to the influence of the gravitational field of the galaxy in the standard picture  $\sim O(\Delta R_s/R^2)$ . As long as  $\Delta^2/R^2 \ll M_\odot/(\gamma m)$ , the effect is small and local distortions of the metric by local objects can be treated as usual. Measurements of the deflection of light by a star or the precession of perihelia of planets would give the standard results. The deflection of light by a distant galaxy (relevant for gravitational lensing) is reduced only if the light passes through the halo. Effects of this type (also for clusters) may perhaps be used for future tests of our speculation. The observation of rotation curves of distant galaxies measures the Doppler shift in frequencies  $\sim R\dot{\varphi} = v_{rot}$  vs. the observation angle<sup>§</sup>  $\psi = R_{min}/d$ , where  $d$  is the distance to the observer and  $R_{min}$  the value of the standard coordinate  $r$  reached at the closest distance of the photon trajectory to the center of the object. Therefore the “measured radius of the orbit” corresponds to the standard coordinate  $r$ , and the function  $A(r)$  plays no role.

The singularity in the center of the galaxy differs from the standard black hole solution. This may have interesting consequences in view of the difficulties of standard cold dark matter galaxies to describe the inner region. Unless  $|\gamma|$  is tiny, the scalar field would be an important ingredient for the galactic dynamics. This would influence many aspects of our present understanding. The two last issues require, however, the inclusion of the baryonic (and dark) matter and also possibly of the scalar potential energy.

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<sup>‡</sup>Note that inside the halo  $\varphi'^2 \sim r^{-2}$  and therefore  $\rho_{grad} \sim r^{-4}$  (eq. (14)).

<sup>§</sup>We assume an observer in flat space. If the observer is within a cosmon lump, the deflection of light by its own “galaxy” is reduced as compared to a “dark matter galaxy”.

We next discuss the effects of the scalar potential. They depend strongly on the sign of  $\gamma$ . We assume that at very large distances the scalar field approaches its cosmological value. If quintessence is important today, this implies  $V(\varphi_\infty) \approx \rho_c = 6M^2H^2$ , with  $H$  the Hubble parameter. In the region of small  $V$  discussed previously the scalar field depends on  $r$  in a simple form

$$\varphi(r) \approx \begin{cases} \varphi_\infty - \frac{2M\epsilon\rho_H}{r} & \text{outside the halo} \\ \varphi_\infty + 2M\epsilon \ln \frac{r}{\rho_H} & \text{inside the halo} \end{cases} \quad (23)$$

where  $\epsilon = \text{sign}(\gamma)$ . For  $\gamma < 0$  the scalar field increases for  $r \rightarrow 0$ . The potential energy therefore decreases and remains completely insignificant for our discussion such that the previous discussion applies without modifications. On the other hand, for  $\gamma > 0$  the potential energy increases for decreasing  $r$  and may finally become important in the inner region of the lump. A criterion for the importance of the potential is the relative size of the dimensionless quantities

$$v = \frac{Ar^2V}{M^2}, \quad y = \frac{\partial \ln B}{\partial \ln r} \quad (24)$$

At large distance gravity becomes very weak (cf. eq. (11)) and  $v$  dominates for  $r > r_{eq}$ ,

$$r_{eq} = (m/8\pi\rho_c)^{1/3} \quad (25)$$

Beyond  $r_{eq}$  the metric and  $\varphi$  are given by cosmology (or the environment at larger scales like clusters). For  $m = (10^{11} - 10^{12})M_\odot$  one finds that  $r_{eq} = (320 - 700)kpc$  is outside the range of interest of this work<sup>¶</sup> (note  $r_{eq} > \rho_H$ ). Outside the halo the change of  $V$  and  $A$  are small,  $v \sim r^2$ , whereas inside the halo region one has

$$v = \frac{r^4\rho_c}{\rho_H^2 M^2} \exp(-\alpha \frac{\varphi(r) - \varphi_\infty}{M}) = 6H^2\rho_H^2 \left(\frac{r}{\rho_H}\right)^{4-2\epsilon\alpha} \quad (26)$$

For  $\gamma > 0$  ( $\epsilon = 1$ ), the potential becomes first negligible for  $r < r_{eq}$  until  $v$  rises again due to the decrease of  $\varphi$ . Then  $v$  becomes comparable to  $y \approx R_s/\rho_H$  at some radius  $r_V$ , provided  $\alpha > 2$ . The scale  $r_V$  depends on  $\alpha$

$$\frac{r_V}{\rho_H} = X_V^{\frac{1}{2\alpha-4}}, \quad X_V = \frac{6H^2\rho_H^3}{R_s} \approx 2 \cdot 10^{-3} \quad (27)$$

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<sup>¶</sup>The relation  $\rho_c = m/(8\pi r_{eq}^3)$  associates  $r_{eq}$  with the average distance between galaxies up to a factor of order one.

where the last number is a guideline for  $m = 3 \cdot 10^{11} M_{\odot}$ ,  $v_{rot} = 150$  km/sec. For  $\alpha \lesssim 3.5$  the ratio  $r_V/\rho_H$  is smaller than 0.1. Numerically we find flat rotation curves with  $v_{rot} \leq 150$  km/sec for  $\alpha$  below or in the vicinity of two. For larger  $\alpha$  the rotation velocities increase as  $r$  decreases. For  $\alpha \gtrsim 2.3$  only the solutions with  $\gamma < 0$  show an interesting halo in this velocity range. For the exponential potential (2) we have found no solution for the gravity-scalar system where the rotation curve decreases in some “inner region”.

For a real galaxy baryonic matter and possibly some form of incoherent dark matter are obviously a crucial ingredient for the dynamics of the inner region. In this context, the incoherent dark matter could be composed of small-size cosmon lumps or consist of so far undiscovered particles. Indeed, cosmon lumps can exist with various sizes and the dynamics of small lumps may be close to a liquid of nonrelativistic particles. Dark matter could also be mimicked by more general small wavelength fluctuations of the cosmon field. Without an understanding of the dynamics of matter in the inner region, it seems premature to decide if the cosmon lump has something to do with a galaxy. The solution presented here could be a valid description of the halo region if galaxies are characterized by large  $|\gamma|$ . In this case new particles are not necessarily needed for dark matter. A perhaps more conservative scenario would assume that some form of dark matter plays an important role in the halo. Then  $|\gamma|$  could be of the order one and smaller. In this event the interplay between the cosmon, gravity and matter may lead to new solutions with an effective halo region dominated by incoherent dark matter. An interesting observational test of the “pure cosmon halo” discussed in this note would compare the mass determinations of galaxies from rotation curves with independent ones from gravitational lensing. For the “cosmon halo” the two estimates need not to coincide.

In summary, we have discussed the isotropic solutions for a scalar field coupled to gravity which are relevant for extended objects in the framework of cosmological scenarios with quintessence. We have not yet investigated the stability properties and the detailed character of the singularity at the origin. Both questions depend on the so far omitted matter in the bulk region. For a substantial cosmon density (large  $|\gamma|$ ) one finds a region with a flat rotation curve. It can fit with the typical scales for mass, size and rotational velocities of a galactic halo. Cosmon lumps should therefore be considered as interesting candidates for galaxies. Again, the detailed form of the rotation curve will be influenced by the matter in the bulk region. More generally, if quintessence plays a role in cosmology, the effects of the cosmon may be relevant for our understanding of extended objects as well!

## References



- [1] C. Wetterich, Nucl. Phys. **B302** (1988) 668; Astron. Astrophys. **301** (1995) 321 [hep-th/9408025]
- [2] P. J. Peebles, B. Ratra, Astrophys. J. Lett. **325** (1988) L17;  
B. Ratra, P. J. Peebles, Phys. Rev. **D37** (1988) 3406
- [3] R. Caldwell, R. Dave, P. Steinhardt, Phys. Rev. Lett. **80** (1998) 1582;  
P. Steinhardt, L. Wang, I. Zlatev, Phys. Rev. **D59** (1999) 12 3504
- [4] P. G. Ferreira, M. Joyce, Phys. Rev. **D58** (1998) 023503
- [5] L. Wang, P. Steinhardt, ApJ **508** (1998) 483
- [6] M. Doran, J. Schwindt, C. Wetterich, astro-ph/0107525
- [7] G. Huey et al., Phys. Rev. **D59** (1999) 063005;  
L. Amendola, Phys. Rev. Lett. **86** (2001) 196;  
P. Corasaniti, E. J. Copeland, astro-ph/0107378
- [8] M. Doran, M. Lilley, J. Schwindt, C. Wetterich, astro-ph/0105306  
(to appear in ApJ);  
M. Doran, M. Lilley, C. Wetterich, astro-ph/0105457
- [9] A. G. Riess et al., Astron. J. **116** (1998) 1009;  
S. Perlmutter et al., ApJ **517** (1999) 565
- [10] C. Wetterich, hep-ph/0108266
- [11] J. Ellis, S. Kalara, K. Olive, C. Wetterich, Phys. Lett. **228B** (1989) 264
- [12] H. A. Buchdahl, Phys. Rev. **115** (1959) 1325;  
M. Wyman, Phys. Rev. **D24** (1981) 839;  
P. Jetzer, D. Scialom, Phys. Lett. **A169** (1992) 12
- [13] C. Wetterich, Nucl. Phys. **B302** (1988) 645
- [14] M. Fitch, S. Tremaine, Ann. Rev. Astron. Astrophys. **29** (1991) 409